BENHA UNIVERSITY
FACULTY OF ENGINEERING AT SHOUBRA

Post-Graduate
ECE-601
Active Circuits

Lecture #5
Microwave Filters

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Agenda

- Introduction
- Microwave Filter Design by the Insertion Loss Method
- Scaling of Low Pass Prototype Filters
- Stepped Impedance Low Pass Filters

INTRODUCTION

Introduction

- A filter is a two-port network used to control the frequency response at a certain point in an RF or microwave system by providing transmission at frequencies within the pass band of the filter and attenuation in the stop band of the filter.
- Typical frequency responses include low-pass, high-pass, band pass, and band-reject characteristics.
- Applications can be found in virtually any type of RF or microwave communication, radar, or test and measurement system.
- The ***image parameter method*** of filter design was developed in the late 1930s and was useful for low-frequency filters in radio and telephony.
- Today, most microwave filter design is done with sophisticated computer-aided design (CAD) packages based on the ***insertion loss method***.

Introduction..

- Filters designed using the *image parameter method* consist of a cascade of simpler two port filter sections to provide the desired cutoff frequencies and attenuation characteristics but do not allow the specification of a particular frequency response over the complete operating range.
- Thus, although the procedure is relatively simple, the design of filters by the image parameter method often must be iterated many times to achieve the desired results.
- The *insertion loss method*, uses network synthesis techniques to design filters with a completely specified frequency response.
- The design is simplified by beginning with low-pass filter prototypes that are normalized in terms of impedance and frequency.
- Transformations are then applied to convert the prototype designs to the desired frequency range and impedance level.

Introduction…

- Both the image parameter and insertion loss methods of filter design lead to circuits using lumped elements (capacitors and inductors).
- For microwave applications such designs usually must be modified to employ distributed elements consisting of transmission line sections.
- The **Richards transformation** and the **Kuroda identities** provide this step.

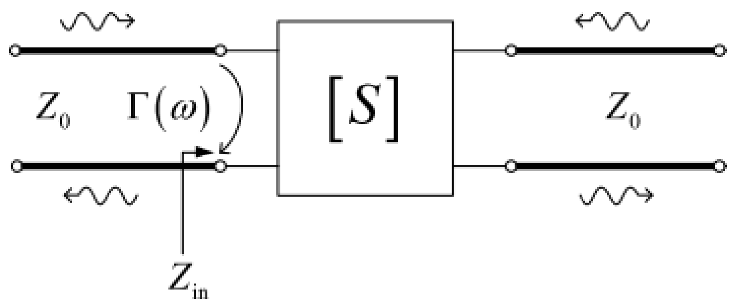
MICROWAVE FILTER DESIGN BY THE INSERTION LOSS METHOD

Insertion Loss Method

- In this technique, the relative power loss due to a lossless filter with reflection coefficient Γ(ω) is specified in the power loss ratio P_{LR} defined as:

$$P_{LR} = \frac{P_{inc}}{P_{load}} = \frac{P_o}{P_o\left[1-|\Gamma(\omega)|^2\right]}$$

$$P_{LR} = \left[1-|\Gamma(\omega)|^2\right]^{-1}$$



- If both the load and source ports are matched for this network, then $P_{LR}=|S_{21}|^2$.

From Section 4.1 we know that $|\Gamma(\omega)|^2$ is an even function of ω; therefore it can be expressed as a polynomial in ω^2. Thus we can write

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2)+N(\omega^2)}, \quad (8.51)$$

where M and N are real polynomials in ω^2. Substituting this form in (8.49) gives the following:

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}. \quad (8.52)$$

Types of Low Pass Filters

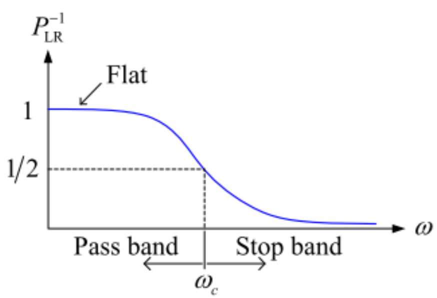
- Four types based on eqn 8.52.

1. Maximally Flat, Butterworth, Binomial Filter:

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^{2N}$$

where $N =$ filter order and $\omega_c =$ cutoff frequency.

If $k = 1$, then $P_{LR} = 2$ at $\omega = \omega_c$, which is the 3-dB frequency:



For large ω and with $k = 1$, then $P_{LR} \approx k^2 \cdot \left(\frac{\omega}{\omega_c} \right)^{2N} = 1^2 \cdot \left(\frac{\omega}{\omega_c} \right)^{2N}$

From this result we learn that the insertion loss IL, defined as

$$\text{IL} = 10 \log \left(P_{LR} \right), \qquad (8.50)$$

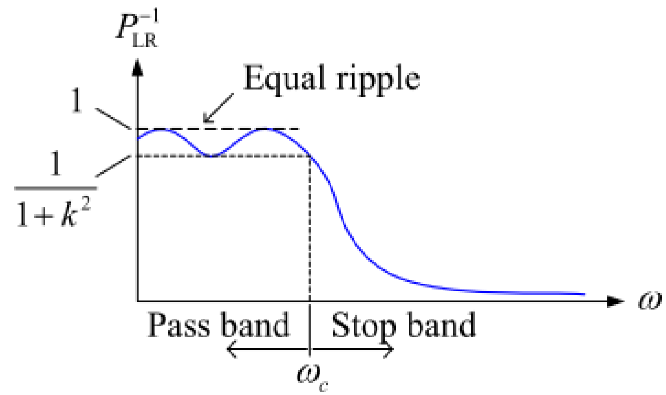
increases by $20N$ dB/decade in the stop band for the maximally flat low pass filter.

Types of Low Pass Filters..

2. Equal Ripple or Chebyshev Filter:

$$P_{LR} = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

where $T_N(x)$ is the Chebyshev polynomial.



- Generally, N is chosen to be an odd integer when the source and load impedances are equal (two-sided filters).
- For large ω/ω_c and using the large argument form of T_N, $P_{LR} \approx \frac{k^2}{4}\left(\frac{2\omega}{\omega_c}\right)^{2N}$
- As with the Butterworth filter, an increase of 20N dB/decade, but with the extra factor $\frac{2^{2N}}{4}$

there is more roll off

$$N = 3 \quad \Rightarrow \quad \log_{10}\left(\frac{2^{2\cdot 3}}{4}\right) = 12.0 \text{ dB}$$

$$N = 5 \quad \Rightarrow \quad \log_{10}\left(\frac{2^{2\cdot 5}}{4}\right) = 24.1 \text{ dB}$$

Types of Low Pass Filters…

3. Elliptic Filter:

- This type of low pass filter has an equi-ripple response in both the pass band and the stop band.
- It has a “faster” roll off than the previous two filters.

4. Linear Phase Filter:

- If it’s important that there be no signal distortion, then the phase of the filter must be linear in the passband.

General Procedure for Filter Design

- The general procedure for designing a filter using the insertion loss method can be summarized in three steps

1. Filter specifications. These include the cutoff frequency, the stop-band attenuation, the pass-band insertion loss, the pass-band behavior, etc.
2. Design a "low pass **prototype**" circuit. In such a prototype, $R = 1\ \Omega$ and $\omega_c = 1$ rad/s. Filter tables are used for this step, or perhaps a computer package.
3. Scale and conversion. Finally, the filter is scaled to the proper impedance level and, if desired, to a high pass, band pass, or band stop topology.

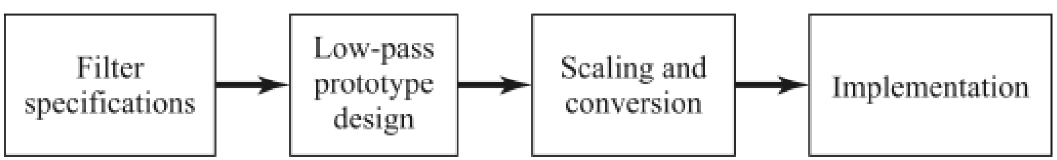
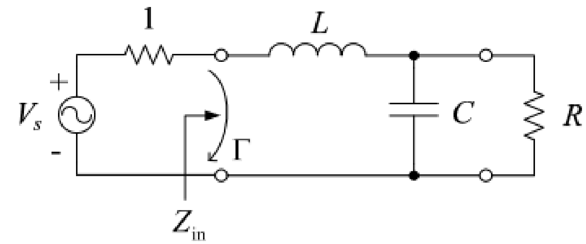


FIGURE 8.23 The process of filter design by the insertion loss method.

Prototype Circuit for the Low Pass Filter

- we'll derive the L and C values for a second order, low pass "prototype" filter:



$$Z_{\text{in}} = j\omega L + \frac{1}{j\omega C} \| R = j\omega L + \frac{R}{1+j\omega RC}$$

$$\Gamma = \frac{Z_{\text{in}} - 1}{Z_{\text{in}} + 1}$$

$$P_{\text{LR}} = \left[1 - \frac{Z_{\text{in}}-1}{Z_{\text{in}}+1} \cdot \frac{Z_{\text{in}}^*-1}{Z_{\text{in}}^*+1}\right]^{-1} = \frac{\left|Z_{\text{in}}+1\right|^2}{2\left(Z_{\text{in}}+Z_{\text{in}}^*\right)} = 1 + \frac{1}{4R}[(1-R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4].$$

For a maximally flat low pass filter,

$$P_{\text{LR}} = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N} \underset{\substack{N=2 \\ \omega_c=1}}{=} 1 + \omega^4 \longrightarrow L = C = \sqrt{2} \quad \text{and} \quad R = 1$$

This circuit is a "prototype" in that

1. The source <u>and</u> load resistances = 1 Ω, and
2. $\omega_c = 1$ rad/s

Prototype Circuit for the Low Pass Filter

The two topologies for low pass prototype circuits are shown

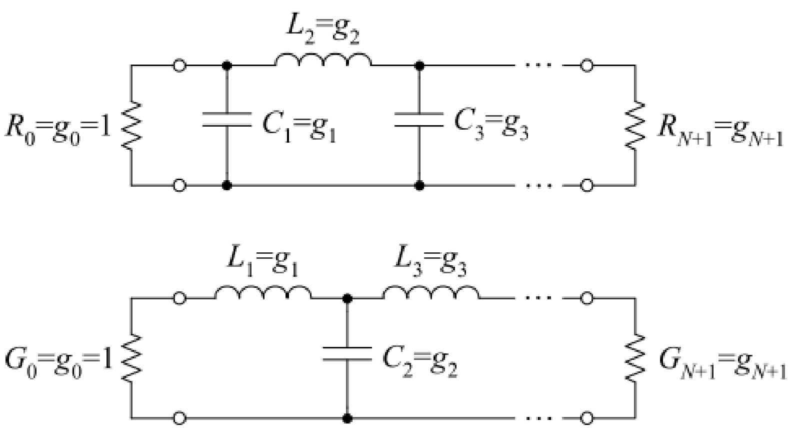


TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

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Both give identical responses. The "immitance" values, g, for a low pass filter are defined as

$$g_k = \begin{cases} \text{inductance for series elements} \\ \text{capacitance for shunt elements} \end{cases} \quad (k = 1, \ldots, N)$$

Filter tables can be used to determine these parameters in prototype circuits for maximally flat, equi-ripple, and other types of filters.

SCALING OF LOW PASS PROTOTYPE FILTERS

Types of Scaling for Low Pass Prototype

- It is possible to *scale and transform* the low pass prototype filter to obtain a low pass, high pass, band pass, and band stop filters for any impedance "level" (R_s=R_L) and cutoff frequency.
- There are two types of scaling for low pass prototype circuits, impedance scaling and frequency scaling:

1. Impedance Scaling:

- Since the filter is a linear circuit, we can multiply all the impedances (including the terminating resistances) by some factor without changing the transfer function of the filter. Of course, the input and output impedances will change.

If the desired source and load impedances equal R_0 , then →

- $X_L' = R_0 X_L = \omega(R_0 L)$. Therefore, $L' = R_0 L$.
- $X_C' = R_0 X_C = -\frac{1}{\omega}\left(\frac{R_0}{C}\right)$. Therefore, $C' = \frac{C}{R_0}$.
- $R_s' = R_0 \cdot 1 = R_0$.
- $R_L' = R_0 \cdot R_L = R_0 R_L$.

Types of Scaling for Low Pass Prototype..

2. Frequency Scaling.

- As defined for the prototype ω_c = 1 rad/s.
- To scale for a different low pass cutoff frequency, we substitute $\omega \to \frac{\omega}{\omega_c}$
- Applying this to the inductive and capacitive reactances in the prototype filter we find
  - $X_L' = \omega L\big|_{\omega \to \frac{\omega}{\omega_c}} = \omega\left(\frac{L}{\omega_c}\right)$. Therefore, $L' = \frac{L}{\omega_c}$.
  - $X_C' = \frac{1}{\omega C}\Big|_{\omega \to \frac{\omega}{\omega_c}} = \frac{1}{\omega}\left(\frac{\omega_c}{C}\right)$. Therefore, $C' = \frac{C}{\omega_c}$.

For a one-step impedance and frequency scaling process →

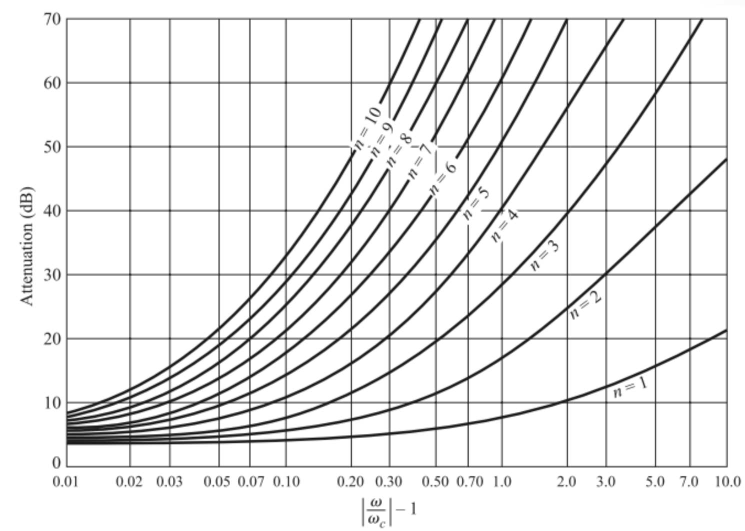
- $L_k' = \frac{R_0 L_k}{\omega_c}$
- $C_k' = \frac{C_k}{\omega_c R_0}$
- $R_s' = R_0$
- $R_L' = R_0 R_L$

Example

- Design a 3-dB, equi-ripple low pass filter with a cutoff frequency of 2 GHz, 50- impedance level, and at least 15-dB insertion loss at 3 GHz.

Solution:

- The first step is to determine the order of the filter needed to achieve the required IL at the specified frequency.

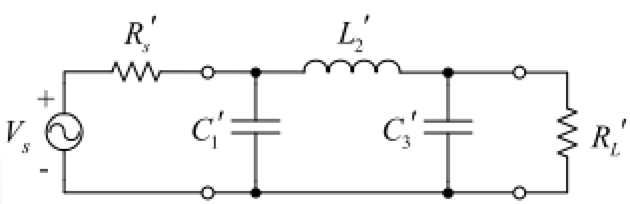


For this filter, we'll choose $N = 3$ to meet the IL specification. From Table 8.4 (3.0-dB ripple), we find the immitance values to be $g_1 = 3.3487$, $g_2 = 0.7117$, $g_3 = g_1$ and $g_4 = 1$.

$$R_0 = 50 \ \ \Omega, \quad f_c = 2 \ \ \text{GHz}$$

then,

- $R_s' = R_L' = R_0 = 50\ \Omega$
- $C_1' = C_3' = \dfrac{C_1}{\omega_c R_0} = \dfrac{g_1}{\omega_c R_0} = \dfrac{3.3487}{2\pi \cdot 2 \cdot 10^9 \cdot 50} = 5.33\ \text{pF}$
- $L_2' = \dfrac{R_0 L_2}{\omega_c} = \dfrac{R_0 g_2}{\omega_c} = \dfrac{50 \cdot 0.7117}{2\pi \cdot 2 \cdot 10^9} = 2.83\ \text{nH}$



3.0 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Example

EXAMPLE 8.3 LOW-PASS FILTER DESIGN COMPARISON

Design a maximally flat low-pass filter with a cutoff frequency of 2 GHz, impedance of 50 Ω, and at least 15 dB insertion loss at 3 GHz. Compute and plot the amplitude response and group delay for $f = 0$ to 4 GHz, and compare with an equal-ripple (3.0 dB ripple) and linear phase filter having the same order.

Solution

First find the required order of the maximally flat filter to satisfy the insertion loss specification at 3 GHz. We have that $|\omega/\omega_c| - 1 = 0.5$; from Figure 8.26 we see that $N = 5$ will be sufficient. Then Table 8.3 gives the prototype element values as

$$g_1 = 0.618,$$
$$g_2 = 1.618,$$
$$g_3 = 2.000,$$
$$g_4 = 1.618,$$
$$g_5 = 0.618.$$

Then (8.67) can be used to obtain the scaled element values:

$$C_1' = 0.984 \text{ pF},$$
$$L_2' = 6.438 \text{ nH},$$
$$C_3' = 3.183 \text{ pF},$$
$$L_4' = 6.438 \text{ nH},$$
$$C_5' = 0.984 \text{ pF}.$$

The final filter circuit is shown in Figure 8.29; the ladder circuit of Figure 8.25a was used, but that of Figure 8.25b could have been used just as well.

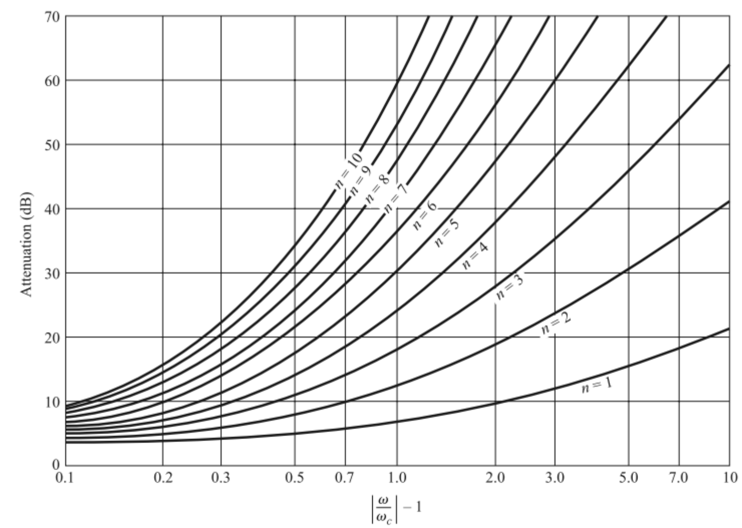


FIGURE 8.26 Attenuation versus normalized frequency for maximally flat filter prototypes.

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

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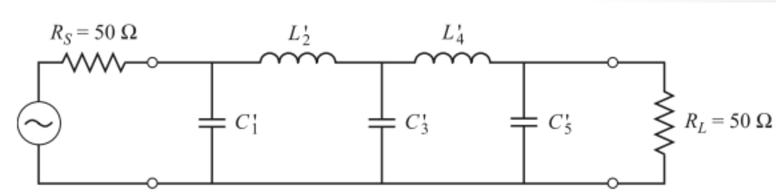
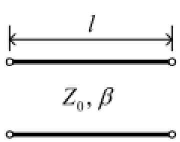


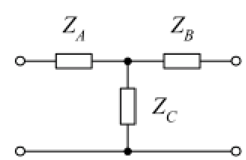
STEPPED IMPEDANCE LOW PASS FILTERS

Filters implementation in microwave circuits

- How do we implement these filters in microwave circuits?
  - Lumped components (such as SMT) can be used up to approximately 5-6 GHz, but their electrical size and the electrical distance between them may not be negligible!
  - Two methods for realizing low pass filters without lumped elements, (1) Stepped impedance and (2) Stubs.
- The equivalent T-network model for a length of TL.



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_0 \sin\beta l \\ jY_0 \sin\beta l & \cos\beta l \end{bmatrix}$$



From Table 4.2

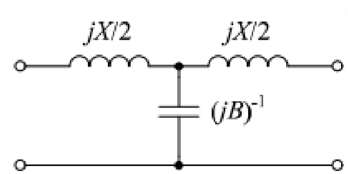
$$Z_{11} = Z_{22} = \frac{A}{C} = -j\frac{Z_0}{\tan\beta l} = -jZ_0\cot\beta l \qquad (8.81a),$$

$$Z_{21} = Z_{12} = \frac{1}{C} = -j\frac{Z_0}{\sin\beta l} = -jZ_0\csc\beta l \qquad (8.81b),$$

$$Z_C = Z_{21} = -jZ_0\csc\beta l$$

$$Z_A = Z_{11} - Z_C = Z_B$$

$$= -jZ_0\cot\beta l + jZ_0\csc\beta l = jZ_0\tan\frac{\beta l}{2}$$



$$\frac{X}{2} = Z_0\tan\left(\frac{\beta l}{2}\right) \text{ and } B = \frac{1}{Z_0}\sin\beta l$$

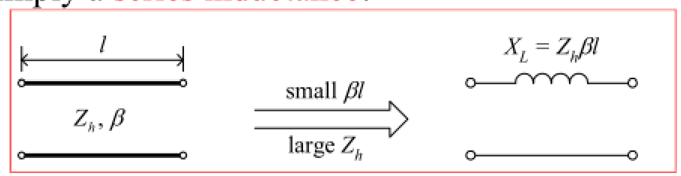
Stepped Impedance Low Pass Filters

- If TL is very short and Z_0 is very large:

$$\frac{X}{2} \approx Z_0 \frac{\beta l}{2} \quad \Rightarrow \quad X \approx Z_0 \beta l$$

$$B \approx 0$$

This is simply a series inductance!

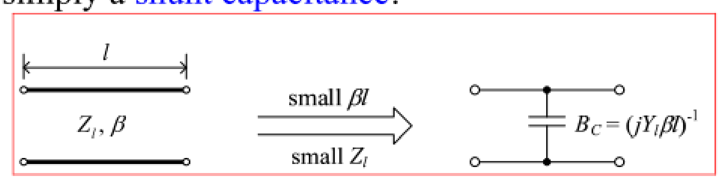


- If TL is short and Z_0 is very small:

$$\frac{X}{2} \approx 0$$

$$B \approx \frac{\beta l}{Z_0} = Y_0 \beta l$$

This is simply a shunt capacitance!



The use of these electrically short high impedance and low impedance sections of TLs is the origin of the alternate name for these stepped impedance filters: Hi-Z, Low-Z filters.

Example

EXAMPLE 8.6 STEPPED-IMPEDANCE FILTER DESIGN

Design a stepped-impedance low-pass filter having a maximally flat response and a cutoff frequency of 2.5 GHz. It is desired to have more than 20 dB insertion loss at 4 GHz. The filter impedance is 50 Ω; the highest practical line impedance is 120 Ω, and the lowest is 20 Ω. Consider the effect of losses when this filter is implemented with a microstrip substrate having $d = 0.158$ cm, $\epsilon_r = 4.2$, $\tan\delta = 0.02$, and copper conductors of 0.5 mil thickness.

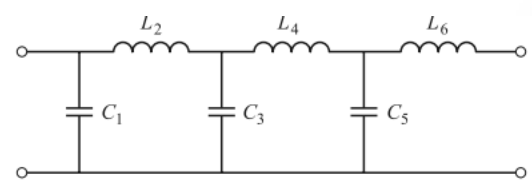
Solution

To use Figure 8.26 we calculate

$$\frac{\omega}{\omega_c} - 1 = \frac{4.0}{2.5} - 1 = 0.6;$$

then the figure indicates $N = 6$ should give the required attenuation at 4.0 GHz. Table 8.3 gives the low-pass prototype values as

$$g_1 = 0.517 = C_1,$$
$$g_2 = 1.414 = L_2,$$
$$g_3 = 1.932 = C_3,$$
$$g_4 = 1.932 = L_4,$$
$$g_5 = 1.414 = C_5,$$
$$g_6 = 0.517 = L_6.$$



Low-pass filter prototype circuit.

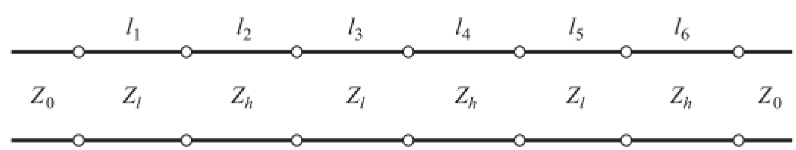
$$\beta\ell = \frac{LR_0}{Z_h} \quad \text{(inductor)} \tag{8.86a}$$

$$\beta\ell = \frac{CZ_\ell}{R_0} \quad \text{(capacitor),} \tag{8.86b}$$

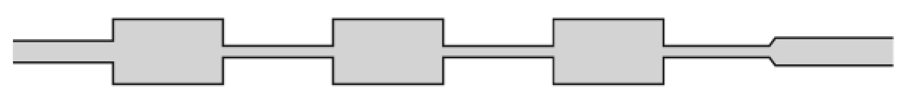
Next, (8.86a) and (8.86b) are used to replace the series inductors and shunt capacitors with sections of low-impedance and high-impedance lines. The required electrical line lengths, $\beta\ell_i$, along with the physical microstrip line widths, W_i, and lengths, ℓ_i, are given in the table below.

Section	$Z_i = Z_\ell$ or $Z_h(\Omega)$	$\beta\ell_i$ (deg)	W_i (mm)	ℓ_i (mm)
1	20	11.8	11.3	2.05
2	120	33.8	0.428	6.63
3	20	44.3	11.3	7.69
4	120	46.1	0.428	9.04
5	20	32.4	11.3	5.63
6	120	12.3	0.428	2.41

The final filter circuit is shown in Figure 8.40b, with $Z_\ell = 20\ \Omega$ and $Z_h = 120\ \Omega$. Note that $\beta\ell < 45°$ for all but one section. The microstrip layout of the filter is shown in Figure 8.40c.



(b)



(c)

Figure 8.41 shows the calculated amplitude response of the filter, with and without losses. The effect of loss is to increase the passband attenuation to about

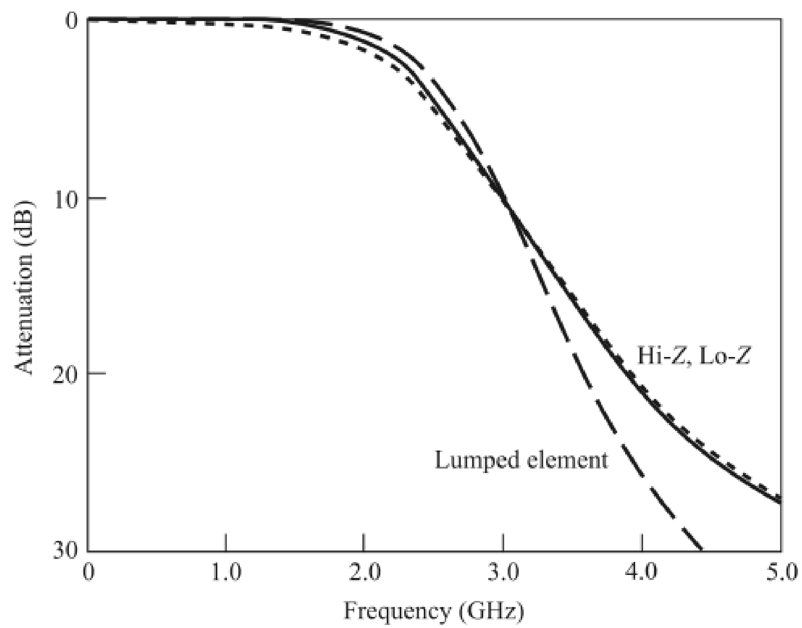


FIGURE 8.41 Amplitude response of the stepped-impedance low-pass filter of Example 8.6, with (dotted line) and without (solid line) losses. The response of the corresponding lumped-element filter is also shown.

1 dB at 2 GHz. The response of the corresponding lumped-element filter is also shown in Figure 8.41. The passband characteristic is similar to that of the stepped impedance filter, but the lumped-element filter gives more attenuation at higher frequencies. This is because the stepped-impedance filter elements depart significantly from the lumped-element values at higher frequencies. The stepped-impedance filter may have other passbands at higher frequencies, but the response will not be perfectly periodic because the lines are not commensurate. ■

- For more details, refer to:
  - Chapter 8, Microwave Engineering, David Pozar_4ed.
- The lecture is available online at:
  - *http://bu.edu.eg/staff/ahmad.elbanna-courses/11983*
- For inquires, send to:
  - ahmad.elbanna@feng.bu.edu.eg